Analysis of the Time Response of Nonuniform Multiconductor Transmission Lines with a Method of Equivalent Cascaded Network Chain

Jun-Fa Mao and Zheng-Fan Li

Abstract—**In this paper nonuniform multiconductor transmission lines are considered to be equivalent to a cascaded chain of many multiport subnetworks which are made of short sections of uniform lines. The *ABCD* matrices of the subnetworks can be obtained by the matrix series expansions of their analytic expressions. As long as the number of the subnetworks is large enough to reflect the line nonuniformity fully, the expansions will converge so fast that a few of low-order series terms will be good approximations. After the overall *ABCD* matrix of the cascaded network chain is evaluated from that of each subnetwork, the time of response of transmission lines can be analyzed. The lines may have frequency-dependent parameters and arbitrary nonlinear terminals. Furthermore, transmission systems with branches uniform and nonuniform transmission lines can be studied with this method conveniently. The analysis accuracy and efficiency are discussed in detail.**

I. Introduction

MULTICONDUCTOR transmission lines are usually used as interconnections in large-scale high-speed integrated circuits. When the signal speed is relatively high, analysis of the time response of such lines becomes necessary to properly design and analyze integrated circuits. Many authors [1]–[7] have studied various kinds of transmission lines with various methods. Relatively speaking, the methods and techniques for uniform transmission lines are more mature and perfect. For nonuniformly coupled transmission lines, papers such as [6], [7] have devoted some efforts to the analysis, but the frequency-dependence of line parameters (such as that of resistance due to the skin effect) wasn't dealt with in these papers. In a recent paper [8], tapered transmission lines were investigated by the method of scattering parameters. The frequency-dependence of scattering parameters due to the change in effective dielectric constant was considered.

In this paper, a method of equivalent cascaded network chain is developed to analyze the time response of nonuniform transmission lines. N-conductor nonuniform lines are considered to be equivalent to a cascaded chain of a series of $2N$-port subnetworks. Each subnetwork is approximated to be made of N-conductor uniform lines with fairly short length. The *ABCD* matrices of these subnetworks can be derived by solving the telegraphers' equations in the frequency-domain, with the solutions in the form of matrix exponentials. To calculate the matrix exponentials, we expanded them into matrix infinite series, then the expansions of the subnetwork *ABCD* matrices are gotten with very simple series terms. From the expansions a useful property of the subnetwork *ABCD* matrices relevant to frequency is proven. As long as the number of subnetworks is large enough to reflect the line nonuniformity, the series will converge so fastly that only a few of low-order series terms are good approximations to the subnetwork *ABCD* matrices. Segmenting the nonuniform lines into many subnetworks has two advantages. One is that it deals with the nonuniformity of lines, another is that it makes the expanded series converge more fastly. After the *ABCD* matrices of all subnetworks have been evaluated, the overall *ABCD* matrix of the cascaded network chain can be obtained simply. Combined with the boundary conditions and the techniques of fast Fourier transform and numerical convolution, analysis of the time response can be made. Nonuniform lines having frequency-dependent parameters and arbitrary terminals can be analyzed in this way. Furthermore, transmission systems with branches of uniform and nonuniform transmission lines can be investigated. The errors and the advantages over some other existing analysis methods are discussed in detail before concluding this paper, which demonstrates that this method of equivalent cascaded network chain is reliable and efficient when applied to analyze nonuniform transmission lines.

II. Theory

Consider the multiconductor transmission line illustrated in Fig. 1. Under the quasi-TEM approximation, it satisfies the telegraphers' equations, which can be written in the frequency-domain as

$$\partial[V(x, w)]/\partial x = -[Z(x, w)]\,[I(x, w)] \tag{1a}$$

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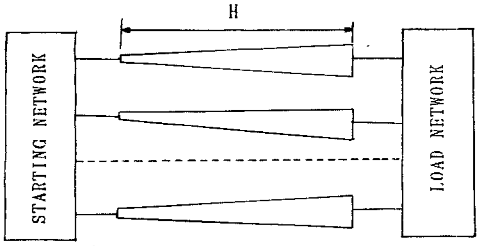


Fig. 1. A nonuniform multiconductor transmission line.

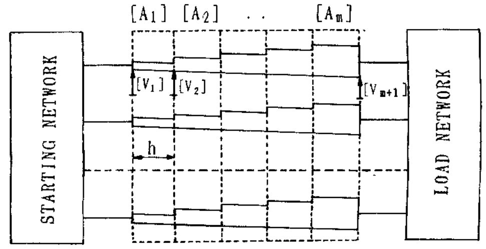


Fig. 2. The equivalent cascaded network chain of a nonuniform line.

$$\partial[I(x, w)]/\partial x = -[Y(x, w)]\,[V(x, w)] \qquad (1b)$$

where $[V(x, w)]$, $[I(x, w)]$ are the column voltage and current vectors at position x and angular frequency w, and

$$[Z(x, w)] = [R(x, w)] + jw[L(x, w)]$$
$$[Y(x, w)] = [G(x, w)] + jw[C(x, w)]$$

with $[L(x, w)]$, $[C(x, w)]$, $[R(x, w)]$ and $[G(x, w)]$ being the inductance, capacitance, resistance and conductance N by N (N is the number of signal conductors) matrices per unit length. For nonuniform lines, (1a) and (1b) almost cannot be analytically solved except for a few special kinds of transmission lines such as single exponential lines. In this chapter the nonuniform transmission lines are considered to be equivalent to a cascaded chain of many subnetworks to get the numerical solutions.

1. Equivalent Cascaded Network Chain of Nonuniform Lines

Let the nonuniform transmission line with a length H in Fig. 1 be uniformly segmented into m sections (see Fig. 2), then the length of each section is $h = H/m$. If m is large enough, all sections can be approximated to be uniform, i.e., $[Z(x, w)]$ and $[Y(x, w)]$ stay constant within every section. Let $[V_i]$, $[I_i]$ ($i = 1, 2, \cdots, m+1$) denote the voltage and current vectors at the ith segmenting point:

$$[V_i] = [V((i-1)h, w)]$$
$$[I_i] = [I((i-1)h, w)].$$

$[Z_i]$ and $[Y_i]$ denote the $[Z(x, w)]$ and $[Y(x, w)]$ at the ith segmenting point. Such segmenting way actually segments the nonuniform lines into m cascaded subnetworks, which are made of short uniform lines. From the theory of network analysis, it is most convenient to calculate the overall network parameters of a cascaded network chain if *ABCD* matrices are taken. If $[A_i]$ is the *ABCD* matrix of the ith subnetwork, $[A]$ is the overall *ABCD* matrix, then

$$\begin{pmatrix}[V_i]\\ [I_i]\end{pmatrix} = [A_i]\begin{pmatrix}[V_{i+1}]\\ [I_{i+1}]\end{pmatrix} \qquad (2a)$$

$$\begin{pmatrix}[V_1]\\ [I_1]\end{pmatrix} = [A]\begin{pmatrix}[V_{m+1}]\\ [I_{m+1}]\end{pmatrix} = \begin{pmatrix}[A1] & [A2]\\ [A3] & [A4]\end{pmatrix}\begin{pmatrix}[V_{m+1}]\\ [I_{m+1}]\end{pmatrix} \qquad (2b)$$

$$[A] = [A_1]\,[A_2] \cdots [A_m] \qquad (3)$$

where $[A1]$, $[A2]$, $[A3]$ and $[A4]$ are matrix partitions of $[A]$. Note that all subnetworks are made of uniform lines, so their *ABCD* matrices can be analytically derived from the solutions of the telegraphers' equations in the frequency-domain [9]:

$$[V_i] = .5(e^{[\beta_i]h} + e^{-[\beta_i]h})\,[V_{i+1}] + .5(e^{[\beta_i]h} - e^{-[\beta_i]h})\,[Z_i^0]\,[I_{i+1}] \qquad (4a)$$

$$[I_i] = .5[Z_i^0]^{-1}(e^{[\beta_i]h} + e^{-[\beta_i]h})\,[Z_i^0]\,[I_{i+1}] + .5[Z_i^0]^{-1}(e^{[\beta_i]h} - e^{-[\beta_i]h})\,[V_{i+1}]$$
$$i = 1, 2, \cdots, m \qquad (4b)$$

where

$$[\beta_i] = ([Z_i]\,[Y_i])^{1/2}$$
$$[Z_i^0] = [\beta_i]^{-1}[Z_i] = [\beta_i]\,[Y_i]^{-1}$$

so we have

$$[A_i] = \begin{pmatrix}[A1_i] & [A2_i]\\ [A3_i] & [A4_i]\end{pmatrix} \qquad (5a)$$

in which

$$[A1_i] = .5(e^{[\beta_i]h} + e^{-[\beta_i]h}) \qquad (5b)$$
$$[A2_i] = .5(e^{[\beta_i]h} - e^{-[\beta_i]h})\,[Z_i^0] \qquad (5c)$$
$$[A3_i] = .5[Z_i^0]^{-1}(e^{[\beta_i]h} - e^{-[\beta_i]h}) \qquad (5d)$$
$$[A4_i] = .5[Z_i^0]^{-1}(e^{-[\beta_i]h} + e^{-[\beta_i]h})\,[Z_i^0]. \qquad (5e)$$

These expressions are accurate for each subnetwork under the uniform approximation. In the next section the matrix exponentials encountered in these expressions are expanded into infinite matrix series to numerically calculate the subnetwork *ABCD* matrices.

2. Calculation of the Subnetwork ABCD Matrices

In the expressions of the subnetwork *ABCD* matrices derived in the above section, there are matrix exponential and matrix square root operations, which are usually redundant. Several methods, such as mode decomposition, Laplace transform, and the method of undetermined coefficients [10], can be used to calculate the matrix exponentials, but these methods are inconvenient and time-consuming to perform on computers, so here we expand the matrix exponentials into matrix infinite series. The matrix square root operation is eliminated as a by-product. Then the expansions of the subnetwork *ABCD* matrices are obtained with very simple series terms. The infinite matrix series of $e^{[\beta_i]h}$ and $e^{-[\beta_i]h}$ are [10]:

$$e^{[\beta_i]h} = 1 + [\beta_i]h + \frac{1}{2!}([\beta_i]h)^2 + \cdots + \frac{1}{k!}([\beta_i]h)^k + \cdots$$

$$e^{-[\beta_i]h} = 1 - [\beta_i]h + \frac{1}{2!}([\beta_i]h)^2 + \cdots + \frac{(-1)^k}{k!}([\beta_i]h)^k + \cdots$$

Inserting these series into expressions (5b)–(5e), and noting $[\beta_i] = ([Z_i][Y_i])^{1/2}$, yield

$$[A1_i] = 1 + \frac{1}{2!}[Z_i][Y_i]h^2 + \cdots + \frac{1}{(2k-2)!}([Z_i][Y_i]h^2)^{k-1} + \cdots = [a_1] + [a_2] + \cdots + [a_k] + \cdots \quad (6)$$

$$[A2_i] = [Z_i]h + \frac{1}{3!}[Z_i][Y_i][Z_i]h^3 + \cdots + \frac{1}{(2k-1)!}([Z_i][Y_i]h^2)^{k-1}[Z_i]h + \cdots \quad (7)$$

$$[A3_i] = [Y_i]h + \frac{1}{3!}[Y_i][Z_i][Y_i]h_i)h^3 + \cdots + \frac{1}{(2k-1)!}[Y_i]h([Z_i][Y_i]h^2)^{k-1} + \cdots \quad (8)$$

$$[A4_i] = 1 + \frac{1}{2!}[Y_i][Z_i]h^2 + \cdots + \frac{1}{(2k-2)!}([Y_i][Z_i]h^2)^{k-1} + \cdots \quad (9)$$

where

$$[a_k] = \frac{1}{(2k-2)!}([Z_i][Y_i]h^2)^{k-1} \qquad k = 1, 2, \cdots.$$

In paper [9] several properties of $[A1_i]$, $[A2_i]$, $[A3_i]$ and $[A4_i]$ are given, the most helpful one for calculating the subnetwork *ABCD* matrices is that $[A1_i] = [A4_i]^T$. Here another userful property is derived:

$$[A_i(w)] = [A1_i(-w)]^* \quad (10a)$$

$$[A2_i(w) = [A2_i(-w)]^* \quad (10b)$$

$$[A3_i(w) = [A3_i(-w)]^* \quad (10c)$$

$$[A4_i(w)] = [A4_i(-w)]^* \quad (10d)$$

i.e., $[A1_i(w)]$, $[A2_i(w)]$, $[A3_i(w)]$ and $[A4_i(w)]$ are the conjugate matrices of $[A1_i(-w)]$, $[A2_i(-w)]$, $[A3_i(-w)]$ and $[A4_i(-w)]$, respectively.

Proof: From the physical point of view, generally we have

$$[R_i(w)] = [R_i(-w)]$$

$$[L_i(w)] = [L_i(-w)].$$

Hence,

$$[Z_i(w)] = [Z_i(-w)]^*$$

similarly,

$$[Y_i(w)] = [Y_i(-w)]^*.$$

According to the property of complex function, there is

$$[Z_i(w)][Y_i(w)] = ([Z_i(-w)][Y_i(-w)])^*$$

so (10a)–(10d) are right from (6)–(9) for $[A1_i]$, $[A2_i]$, $[A3_i]$ and $[A4_i]$.

The matrix infinite series (6)–(9) are absolutely convergent, and their convergence rates are in relation with the magnitude of each eigenvalue ($|\lambda_{i,k}|$, $i = 1, 2, \cdots, m$, $k = 1, 2, \cdots, N$) of the product of $[Z_i][Y_i]h^2$. The smaller the magnitude of $\lambda_{i,k}$, the better the convergence rates. From the definition of eigenvalue:

$$[Z_i][Y_i]h^2[e_{i,k}] = \lambda_{i,k}[e_{i,k}] \quad (11)$$

where $[e_{i,k}]$ is the eigenvector corresponding to $\lambda_{i,k}$, it can be seen that for given transmission lines, i.e., for given product of $[Z_i][Y_i]$, larger number (m) of subnetworks will produce smaller $|\lambda_{i,k}|$ and better convergence rates. Actually, as long as the number (m) of subnetworks is large enough (h is small enough) to reflect the line nonuniformity fully, these matrix series will converge so fast that only a few of low-order series terms may be good approximations to $[A1_i]$, $[A2_i]$, $[A3_i]$ and $[A4_i]$. For example $\Sigma_{j=1}^{k}[a_j]$ may be good approximation to $[A1_i]$ even if $k = 3$. As a special case, if only the first terms of the matrix series (6)–(9) are retained, then this is just the well known finite-difference approximation.

From the above discussion, segmenting the transmission lines into many cascaded subnetworks has two advantages, one is that the nonuniformity of lines can be dealed with, another is that the convergence rates of the expansions of the subnetwork *ABCD* matrices become better. There is something to be pointed out here. The segmentation of lines here in this paper has a important difference from that in the characteristics method [7]. In the characteristics method, after the decoupling procedure, the effective electric length of each decoupled single signal conductor becomes different because of the different transmission mode velocities of the transmission system. Furthermore, the segmentation is restricted by the value of time steps. This makes it difficult to segment each decoupled conductor into integer number of sections at the same time, and suitable rounding off must be taken. This will introduce errors to the time response analysis of transmission lines, especially of quiescent lines. While the segmentation in the method in this paper is carried out independently, the number of time samples and the length of time steps have no influence on it, so this problem doesn't arise. On the other hand, unlike the finite-difference method, several low-order series terms (not the first term only) are retained here, so the number of subnetworks is unnecessarily very large to get high accuracy.

After the *ABCD* matrices of all subnetworks have been obtained, (3) is then used to calculate the overall *ABCD* matrix of the cascaded network chain which is the equivalent of the original transmission line to be analyzed. With the boundary conditions known, time response analysis can be performed as follows.

3. Analysis of the Time Response of Transmission Lines

Noting that $[V_1] = [V(0, w)]$, $[I_1] = [I(0, w)]$, $[V_{m+1}] = [V(H, w)]$ and $[I_{m+1}] = [I(H, w)]$, from (2b) there are

$$[V(0, w)] = [A1]\,[V(H, w)] + [A2]\,[I(H, w)] \quad (12a)$$

$$[I(0, w)] = [A4]\,[I(H, w)] + [A3]\,[V(H, w)]. \quad (12b)$$

Equations (12a) and (12b) combined with the boundary conditions are enough to carry out the time response analysis. For linear terminals, the boundary conditions can be expressed in the frequency-domain:

$$[V(0, w)] = [E(w)] - [Z1(w)]\,[I(0, w)] \quad (13a)$$

$$[V(H, w)] = [Z2(w)]\,[I(H, w)] \quad (13b)$$

in which $[Z1(w)]$ and $[Z2(w)]$ are the Z-parameters of the starting network and load network respectively, $[E(w)]$ is the Fourier transform of source voltage vector. After the analysis at each discrete sample frequency is performed, the time response can be obtained by the inverse fast Fourier transform (IFFT). For nonlinear terminals, the boundary conditions are generally given in the time-domain:

$$[V(0, t)] = f_1([I(0, t)] + [E(t)] \quad (14a)$$

$$[V(H, t) = f_2([I(H, t)]) \quad (14b)$$

in which $f_1(\cdot)$ and $f_2(\cdot)$ indicate two nonlinear functions. Inversely Fourier transforming equations (12a) and (12b) yields

$$[V(0, t)] = [A1(t)] * [V(H, t)] + [A2(t)] * [I(H, t)] \quad (15a)$$

$$[I(0, t)] = [A4(t)] * [I(H, t)] + [A3(t)] * [V(H, t)] \quad (15b)$$

where $[A1(t)]$, $[A2(t)]$, $[A3(t)]$ and $[A4(t)]$ are the inverse Fourier transforms of $[A1(w)]$, $[A2(w)]$, $[A3(w)]$ and $[A4(w)]$ which can be obtained by the technique of IFFT, "*" denotes a convolution, for example:

$$[A1(t)] * [V(H, t)] = \int_0^t [A1(t - \tau)]\,[V(H, \tau)]\,d\tau$$

in which the integration can be approximated by a summation when calculated on computers. The system of equations (14a), (14b), (15a) and (15b) is nonlinear and can be solved by the Newton–Raphson method marching-on-in-time to obtain the time response $[V(0, t)]$, $[I(0, t)]$, $[V(H, t)]$ and $[I(H, t)]$. After the waveforms at the two terminal ends of transmission lines are obtained, the response at any segmenting points within the two ends can be evaluated, which is easy to see from Fig. 2.

Uniform transmission lines can also be analyzed by this method, but the most economical way is to consider a whole transmission line to be one subnetwork and evaluate the *ABCD* matrix with the mode decomposition method. As to complicated transmission systems with branches of uniform and nonuniform lines, both the methods can be used, i.e., the nonuniform lines are segmented into a series of cascaded subnetworks the ABCD matrices of which can be obtained through the expansions of matrix exponentials, while the uniform lines are equivalent to only one subnetwork the *ABCD* matrix of which can be evaluated by the mode analysis. Thus much CPU time will be saved. After the ABCD matrix of each branch is obtained, the overall *ABCD* matrix of the transmission system can be evaluated according to the connecting mode. Combined with the boundary conditions at the terminal ends of the branches, time response analysis can be performed in a similar way to that described above.

III. Numerical Examples

Example 1: A Linearly Loaded Two-Conductor Nonuniform Line (Fig. 3)

The transmission system in this example is similar to that described in [6]. The line parameters are

$$l(x) = 387/(1 + k(x))$$

$$l_m(x) = k(x)\,l(x)$$

$$c(x) = 104.13/(1 - k(x))$$

$$c_m(x) = -k(x)\,c(x)$$

$$k(x) = .25(1 + .6 \sin(\pi x + \pi/4))$$

$$r(x) = 1.2\left(\frac{|w|}{2\pi}\right)^{1/2}, \quad r_m(x) = 0$$

$$g(x) = g_m(x) = 0$$

where $l(x)$, $c(x)$, $r(x)$ and $g(x)$ are the diagonal elements of the 2-by-2 inductance, capacitance, resistance and conductance matrices respectively, the unit of which are nH/m, pF/m, Ω/m and nS/m; $l_m(x)$, $c_m(x)$, $r_m(x)$ and $g_m(x)$ are the corresponding second diagonal elements. The unit of w is MHz. The crosstalk voltage at the middle point of the quiescent line given in Fig. 4 compares favorably to that given in [6], which ensures the reliability of this method.

Example 2: A Nonlinearly Loaded Three-Conductor Nonuniform Line

The crossectional view and schematic of the transmission system in this example are given in Fig. 5. There have been many papers published for approaching the distributed parameters for multiconductor transmission lines [11], [12], and here the method and program given in [13]

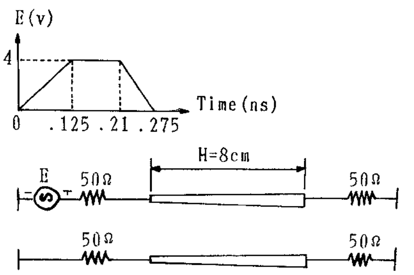


Fig. 3. The transmission system in example 1.

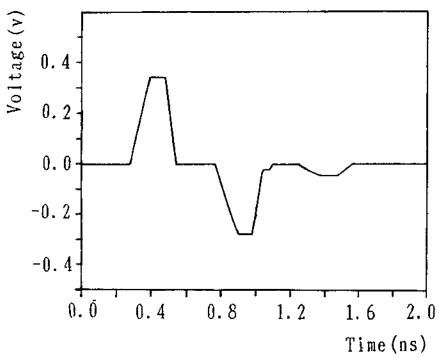
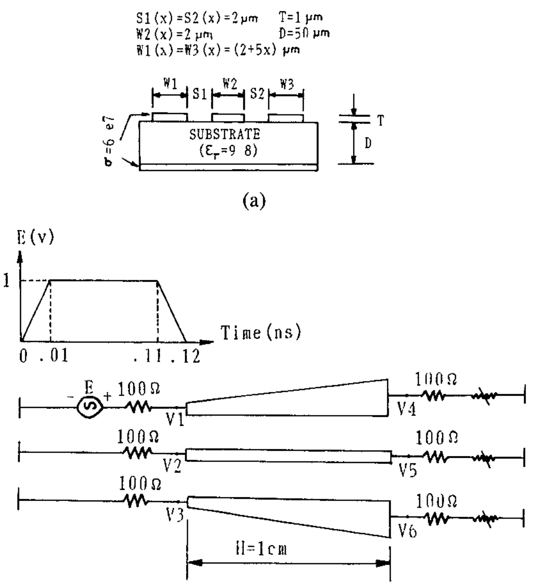


Fig. 4. The crosstalk voltage at the middle of the quiescent line in example 1.



(b)

Fig. 5. The cross sectional view (a) and schematic (b) of the transmission system in example 2.

is used. Because the transverse sections of the signal conductors in this system are fairly small, the resistances are approximated to be the dc values and independent of frequency. So are the other parameters. The nonlinear loads are characterized by the relation:

$$i = 10(e^{40v} - 1)$$

where i is the current in nA following through the loads

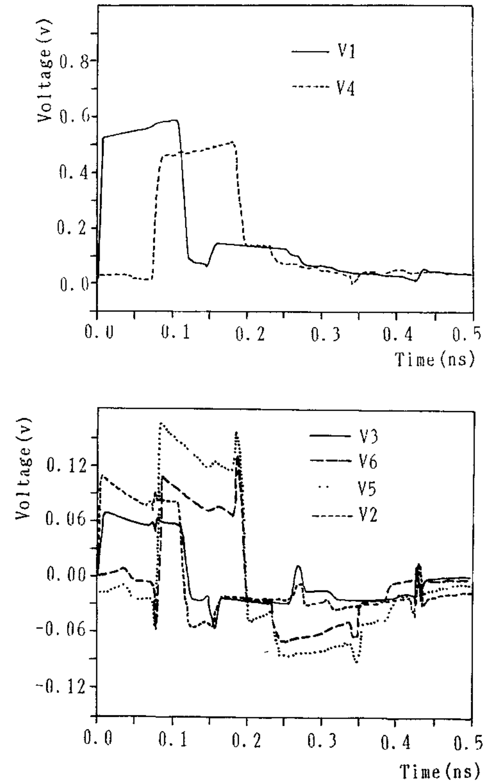


Fig. 6. The response voltages in example 2.

and v is the corresponding drop in V. Fig. 6(a) and (b) show the response obtained with this method.

Example 3: A Transmission System with Branches of Uniform and Nonuniform Lines

The schematic of the transmission system in this example is given in Fig. 7. It includes two branches of uniform lines and one branch of nonuniform line. The nonuniform line is just that in example 1, and the two uniform line branches have the same parameters:

$$[L] = \begin{bmatrix} 309 & 21.7 \\ 21.7 & 309 \end{bmatrix} \text{nH/m},$$

$$[C] = \begin{bmatrix} 144 & -6.4 \\ -6.4 & 144 \end{bmatrix} \text{pF/m}$$

$$[R] = \begin{bmatrix} 524 & 33.9 \\ 33.9 & 524 \end{bmatrix} \cdot \left(\frac{|w|}{2\pi}\right)^{1/2} \text{m}\Omega/\text{m},$$

$$[G] = \begin{bmatrix} 905 & -11.8 \\ -11.8 & 905 \end{bmatrix} \cdot \frac{|w|}{2\pi} \text{nS/m}$$

in which w is still in MHz. Fig. 8(a) and (b) show parts of the response waveforms. Note that the voltages ($V5$ and $V6$ in figure) at the end of branch #3 are very small in magnitude. This can be understood as that the inductances connected in series in this branch are unfavorable for high speed signals to transmit.

In the above three examples, the numbers of frequency

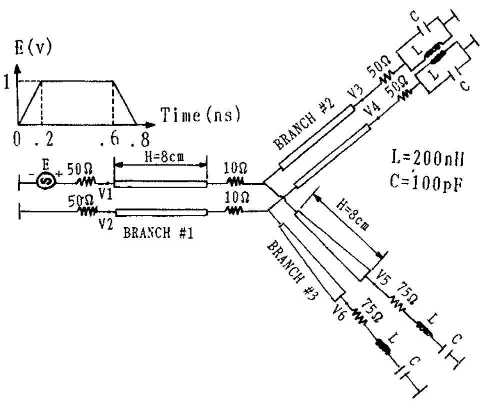


Fig. 7. The transmission system in example 3.

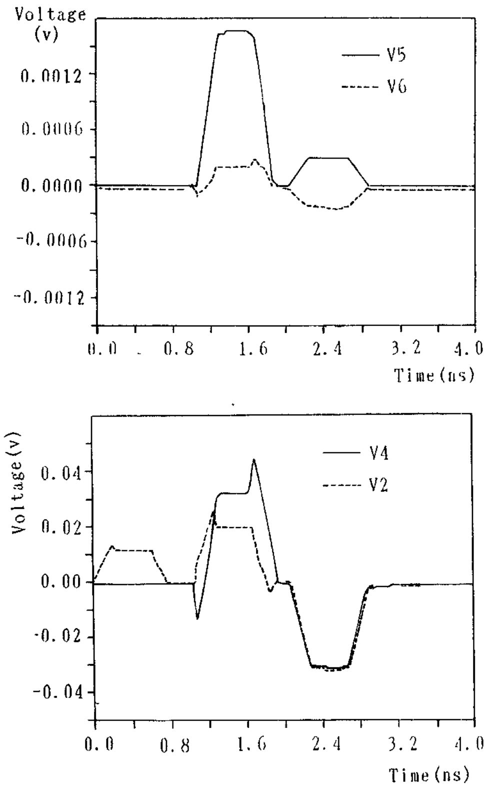


Fig. 8. The response voltages in example 3.

and time samples are all 256, the numbers of low-order series terms taken are all 4, and the numbers of cascaded subnetworks for nonuniform lines are 80, 60, and 80 respectively. On a computer of PC-386/20, the CPU times of these examples are all a few minutes.

IV. Error and Calculation Efficiency Analysis

Apart from the aliasing errors introduced by the finite sample points for FFT and IFFT, there are two factors in this method which may bring errors to the transient analysis, one is the approximation that all subnetworks are made of uniform lines, another is the fact that only a few terms of the matrix infinite series are taken. Obviously, the more the terms of series taken, the more accurate the evaluation of the subnetwork *ABCD* matrices. However, if we enlarge the number (m) of subnetworks, three things will occur. One is that the uniformity approximation for each subnetwork will be more reasonable; the second is that the convergence rates of the expansions of matrix exponentials will be better, so the evaluation of the subnetwork *ABCD* matrices will become more accurate with the same number of series terms taken; the last is that the number of times of error accumulation will be larger. Application practices show that the overall result of these three factors is that the analysis accuracy will be higher with more subnetworks if a suitable number of *low-order* series terms is retained. So there are two ways for getting a given accuracy: enlarging the number of subnetworks and enlarging the number of low-order series terms taken. But under the presupposition that the number of subnetworks is large enough to reflect the line nonuniformity fully, enlarging the number of low-order series terms is more efficient to save the CPU time, which can be seen from the expansions (6)–(9). As a criterion, m is usually chosen to be only large enough to reflect the line nonuniformity (i.e., further enlarging the value of m will not change the results), then necessary number of series terms is taken to get high accuracy. This is the main advantage of the method in this paper over the finite-difference approximation where very large m is required to get high accuracy, and sometimes divergent results may occur due to the too many times of error accumulation, especially when the terminals are nonlinear.

Differing from the mode analysis method or the scattering parameter method, the procedure of mode decomposition in the frequency-domain is omitted here in this method by introducing the *ABCD* matrices of transmission lines, so there is no necessity of tracing the incident and reflective waves, which makes the time response analysis procedure more simple. As mentioned before, the matrix exponentials in the expressions of the subnetwork *ABCD* matrices can also be calculated by the mode decomposition method. In order to ensure the reliability and advantage in computation speed of (6)–(9). One uniform transmission line in example 3 (branch #1 or branch #2) is considered. The *ABCD* matrix of this line is accurately computed by the mode decomposition method using (5b)–(5e), and numerically computed by segmenting this line into 80 sections. Four terms of the infinite matrix series in (6)–(9) are taken when computing the ABCD matrix of a section. Comparison of the results is given in Table I. Note that t_1 and t_2 in the table are comparable because the computation time of the mode decomposition method is independent of the line length. Transmission lines with larger N have also been studied and similar results are obtained (both t_1 and t_2 are proportional to N^3). The table also shows that the lower the frequencies, the less the errors for the *ABCD* matrices. This property is beneficial for improving the analysis accuracy, because for general signal pulses, the magnitudes of the components at lower frequencies are larger than that at higher frequencies.

TABLE I
COMPARISON OF THE *ABCD* MATRIX VALUES AND COMPUTATION TIMES

Frequency (GHz)	*ABS*1		*ABS*2		$1000\ t_2$ (second)	$1000\ t_1$ (second)
	Appr.	Accu.	Appr.	Accu.		
5	0.1832	0.1832	38.95	38.95	3.79	17.14
10	0.3746	0.3746	37.10	37.10	3.79	16.21
15	0.06819	0.06820	3.260	3.261	3.79	16.26
20	0.6201	0.6201	25.66	25.67	3.79	17.13
25	0.8200	0.8221	19.33	19.39	3.79	16.37
30	0.1891	0.1960	1.793	1.762	3.79	17.13

*ABS*1—the magnitude of $[A1(1, 2)]$; *ABS*2—the magnitude of $[A2(2, 2)]$; t_1—the computation time for $[A1]$ and $[A2]$ in the mode decomposition method; t_2—the computation time for $[A1_i]$ and $[A2_i]$ of a section in the numerical method.

V. CONCLUSION

Analysis of the time response of nonuniform transmission lines is carried out by segmenting the transmission lines into a series of cascaded subnetworks in this paper. All subnetworks are approximated to be made of short sections of uniform lines, the *ABCD* matrices of which can be obtained from the expansions of their analytic expressions derived from the solutions of telegraphers' equations in the frequency-domain. There are two advantages in segmenting the lines: one is that the nonuniformity of lines can be dealt with; another is that the expanded matrix infinite series can converge faster. As long as the number of subnetworks is large enough to reflect the line nonuniformity, a few of low-order series terms will be good approximations to the subnetwork *ABCD* matrices. A wide range of transmission systems can be analyzed by this method with little CPU time and satisfactory accuracy.

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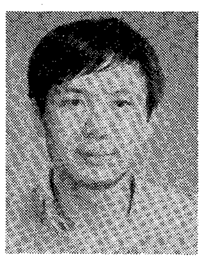

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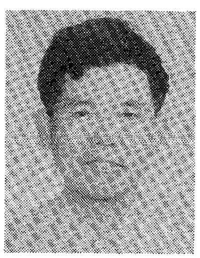

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